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Experimental investigation on rogue waves and their impacts on a vertical cylinder using the Peregrine breather model

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This paper presents an experimental investigation on the rogue waves and the corresponding wave forces acting on a vertical cylinder. The waves are modelled using the Peregrine breather solution of the third-order nonlinear Schrödinger equation. The experimental wave elevations are compared with the theoretical solutions, and the behaviours of wave energy distribution and evolution are calculated using the fast Fourier transformation and wavelet transform (WT) methods. The resulting wave forces acting on the vertical cylinder are compared with the numerical results based on potential flow calculations. Moreover, coherence analyses are conducted for the rogue waves and their impact forces acting on the cylinder based on the WT.

Keywords: rogue waves; Peregrine breather model; wave impacts; vertical cylinder

1. Introduction

Rogue waves, a typical kind of giant asymmetrical transient ocean wave, are believed to be responsible for a great number of severe damages to offshore structures and ships (Lemire 2005; Bertotti and Cavaleri 2008; Kharif et al. 2009). The observations, registrations and accidents about rogue waves are not rare, and many studies on the formation mechanisms of rogue waves have been reported. Chien et al. (2002) catalogued the formation mechanisms of rogue waves into two types: one is due to the impacts of environmental changes and the other is due to the wave–wave interactions. To investigate the rogue wave kinematics and their impacts on offshore structures, it is of great importance to generate rogue waves in physical or numerical wave tanks. Most of the available wave generation models were developed based on the linear dispersive focusing (Longuet-Higgins 1974; Tromans et al. 1991; Cassidy 1999; Kriebel and Alsina 2000), and some of them included the second-order corrections (Ning et al. 2009) or the iterative optimisation of phase distributions (Chaplin 1997; Clauss and Klein 2011). The linear focusing theory works quite efficiently in generating giant waves. However, it cannot well explain the reason why rogue waves occur frequently. The nonlinear self-focusing is regarded as a likely sophisticated formation mechanism of rogue waves and has received more and more attentions (Kharif and Pelinovsky 2003). The modulational instability has been widely studied and is believed to result in a high probability of the occurrence of rogue waves (Onorato et al. 2001; Onorato et al. 2006). The nonlinear Schrödinger (NLS) equation is frequently used to describe the slow modulation of wave envelopes of the carrier waves. It is integrable and many of its analytical solutions are given in explicit expressions (Zakharov 1972). Among them, the Peregrine breather solution is often considered to be the most likely prototype for rogue waves since it is localised in both time and space (Akhmediev et al. 2009; Shrira and Geogjaev 2010).

Many experimental investigations of breather solutions were conducted in recent years. Chabchoub et al. (2011) first presented the experimental results of breather solutions with the observations of the Peregrine soliton in a water wave tank. Onorato et al. (2013) adopted the Peregrine breather solution to study the interaction between rogue waves and a scaled chemical tanker in a wave tank. Besides the experimental studies, Peric et al. (2015) carried out numerical investigations of the Peregrine breather dynamics up to the initial stages of wave breaking and paid attentions to the sub-surface flow field. Hu et al. (2015) performed a series of simulations on the rogue waves based on the Peregrine breather solution in a numerical wave tank.

The wave forces on cylinders have been widely studied and many methods have been proposed to calculate the wave forces on a cylinder (Morison et al. 1950; MacCamy and Fuchs 1954; Faltinsen et al. 1995; Rainey 1995; Chaplin et al. 1997; Abbasnia and Ghiasi 2014). However, it should be noted that most of these studies are limited to moderate waves. For rogue waves, some efforts have been done to characterise the wave forces on a cylinder. Sundar et al. (1999) studied the dynamic pressure distribution around the inclined cylinder extreme waves. Kim and Kim (2003)
investigated the horizontal wave forces on a vertical truncated cylinder under extreme waves and found that the forces are much larger than those calculated using an equivalent-sized laboratory Stokes fifth-order wave. Ma et al. (2009) investigated the higher harmonic forces due to wave focusing on a vertical cylinder and the results show that, with very steep wave crests, the wave force amplitudes at fourth- and fifth-order harmonics are significant. Li et al. (2014) investigated the wave forces caused by multi-directional focused waves on a vertical cylinder. To our best knowledge, all of the studies on the rogue wave forces acting on a vertical cylinder are based on the linear wave focusing. However, investigations on the wave forces on a vertical cylinder under the Peregrine breather are lacking and this is the main motivation for this study.

In this paper, we present the analysis of rogue waves and the corresponding wave forces acting on a truncated cylinder in a wave flume. The Peregrine breather solution model is used to generate rogue waves in the wave flume. To focus on the wave forces due to nonlinear waves, the wave steepness of carrier wave is set to be slightly beyond the breaking threshold. Both the wave elevation and wave forces results are compared with the analytical solutions and their energy distributions are analysed with both fast Fourier transformation (FFT) and wavelet transform (WT) methods. Moreover, coherence analyses have been conducted to reveal the coherence between rogue waves and the corresponding wave forces based on the WT methods.

2. Generation of rogue waves

2.1. The nonlinear Schrödinger equation and the Peregrine breather solution

Under the hypothesis of small amplitudes and quasimonochromatic, the temporal and spatial dynamics of deep-water wave packets could be described by the following form of the NLS equation (Zakharov 1968):

\[ i \left( \frac{\partial A}{\partial t} + c_s \frac{\partial A}{\partial x} \right) - \frac{\omega_0}{8k_0^2} \frac{\partial^2 A}{\partial x^2} - \frac{\omega_0 k_0^2}{2} |A|^2 A = 0, \]

where \( k_0 \) and \( \omega_0 \) represent the wave number and the wave frequency of carrier wave, respectively. \( A(x, t) \) is the complex wave packet, which propagates with the group velocity \( c_s = \omega_0 / 2k_0 \) for deep-water waves. It is crucial that the dimensionless water depth parameter \( k_0 d \) must be larger than 1.36 (Clauss et al. 2011), in order to focus the wave energy through the modulational instability. The surface elevation is given by

\[ \eta(x, t) = \text{Re}(A(x, t) \exp[i(k_0 x - \omega_0 t)]). \]

By using the rescaled variables, the dimensionless form of the NLS can be simplified as

\[ iq_T + q_{XX} + 2|q|^2 q = 0, \]

where

\[ T = -\frac{\alpha_2^2 k_0^2 \omega_0}{4}, \quad X = \sqrt{2} k_0^2 \left( x - \frac{\omega_0}{2k_0^2} t \right), \quad q = i \frac{A}{\omega_0}. \]

Ma (1979) gave a time-periodic breather solution as follows, which tends to be a plane wave as \( X \to \pm \infty \)

\[ q_p(X, T) = \frac{\cos(2 \sin h(2\beta)T - 2i\beta) - \cos(\beta \cos(2 \sin h(\beta)X)) e^{i/\beta}}{\cos(2 \sin h(2\beta)T) - \cos(\beta \cos(2 \sin h(\beta)X))}. \]

On the basis of Ma’s work, Peregrine (1983) gave the first-order rational solution of the NLS equation, which can be understood as the limiting case of the Ma’s solutions when \( \beta \to 0 \),

\[ q_p(X, T) = \left( 1 - \frac{4(1 + 4iT)}{1 + 4X^2 + 16T^2} \right) e^{i/\beta}. \]

Figure 1 presents a dimensionless form of the Peregrine solution, in which the maximum amplitude is three times of the background carrier wave. It is observed that the Peregrine breather is localised both temporally and spatially. Given that the Peregrine breather solution is very similar to the rogue waves, which are characterised by extremely high wave crests and sudden appearance and disappearance; it is widely adopted for modelling rogue waves.

![Figure 1. Peregrine breather solution. (This figure is available in colour online.)](image-url)
The Peregrine breather solution of Equation (1) can be obtained by combining Equations (4) and (6) and given by

\[
A(x,t) = a_0 \exp \left( -i \frac{k_0^2 a_0^2 \omega_0}{2} t \right) \\
\times \left( -1 + \frac{4(1 - i k_0^2 a_0^2 \omega_0 t)}{1 + [2 \sqrt{2 k_0^2 a_0(x + c_s t)}]^2 + k_0^4 a_0^5 \omega_0^4 t^2} \right).
\]

(7)

2.2. Implementation of the Peregrine breather in the wave flume

As a preliminary step, the wave elevations at the wave maker position need to be determined as the boundary conditions. According to Equation (7), the wave elevation at the wave maker position can be given as

\[
\eta(x_{\text{waveraker}}, t) = \text{Re}(A(x_{\text{wave maker}} - x_c, t - t_c)) \\
\times \exp(ik_0(x_{\text{wave maker}} - x_c) - \omega_0(t - t_c)).
\]

(8)

To implement the Peregrine breather in a wave flume, it is vital to determine the exact wave maker displacement, by which the water motion at the wave-maker boundary should be consistent with the initial conditions given by the Peregrine breather solution. It seems extremely hopeless to meet this strict condition. However, when the flap maker displacement was approximately chosen to be proportional to the theoretical surface elevation at the wave maker location, Chabchoub et al. (2011) achieved the wave dynamics very close to the analytical predictions. In this study, the time histories of flap maker motion are determined as follows:

\[
S(t) = \frac{1}{T(\omega_0, l)} \eta(x_{\text{waveraker}}, t).
\]

(9)

where \(T(\omega_0, l)\) is the transfer function between the stroke of flap motion and the wave height based on the potential flow calculation, \(\omega_0\) is the wave frequency of carrier wave and \(l\) is the immersed depth of flap maker. Besides, to avoid large unstable disturbance at the starting time of wave generation, a Sine fade-in for the first 5 s is applied.

3. Experimental set-up and wave parameters

The physical experiment was carried out in the wave flume of State Key Laboratory of Ocean Engineering (SKLOE) in Shanghai Jiao Tong University. The wave flume is 20.0 m long, 1.0 m wide and the standard water depth is 0.9 m. A flap paddle is equipped to generate various kinds of waves and an absorption wave beach stands at the downstream end of the flume to eliminate wave reflection.

Figure 2 shows the sketch of the test set-up. The reference point for the wave calibration and vertical cylinder is 7.0 m away from the wave maker position. To measure the wave evolution, a total of six wave gauges were arranged along the centreline of the flume, i.e., 2, 5, 6, 7, 8 and 9 m away from the wave maker, respectively. A vertical cylinder of 0.4 m draft and 0.1 m in diameter was installed at the reference position and the original wave gauge was then moved to the left of the cylinder, being 0.2 m away from the reference point. The cylinder was mounted stiffly below a rigid support frame and a six-component force/moment transducer between them was used to measure the wave forces. The natural period of the vertical cylinder is 47 Hz, much larger than three times the carrier wave frequency.

In the Peregrine breather model, the only free parameter is the initial steepness \(\varepsilon_0 = k_0 a_0\). Onorato et al. (2013) found that initial steepness larger than 0.1 might probably produce a wave breaking which is of spilling type and then becomes of the plunging type as the initial steepness increases. In the research of Slunyaev et al. (2013), the breaking onsets of numerical simulation and laboratory experiment are \(\varepsilon_0 = 0.095\) and \(\varepsilon_0 = 0.12\) for the first-order rational solution, respectively. In this study, to focus on the wave forces under the nonlinear rogue waves, the wavelength of carrier wave was set to 1.0 m and the background amplitude to 0.02 m. Thus, the initial steepness being 0.126 is slightly beyond the breaking threshold. The wave group velocity is 0.625 m/s and the dimensionless water depth \(k_0 d = 5.65\).

![Figure 2. Sketch of the wave flume experimental set-up. (a) Plan view; (b) elevation view. (This figure is available in colour online.)](image-url)
4. Results and discussions

4.1. Peregrine breather in the wave flume

Before the cylinder was installed in the wave flume (Figure 2), the rogue waves based on the Peregrine breather model were calibrated according to the above-mentioned methodology. Figure 3 presents the wave elevations measured at six selected locations, showing the formation process of Peregrine breather. It can be seen that these wave sequences are similar to regular waves except the disturbance regions and the measured maximum crest grew up gradually as the wave propagated towards the focal location $x = 7.0$ m. At the focal location, the highest maximum wave crest was observed, which is much higher than the maximum wave crest at $x = 2.0$ m.

Figure 4 presents the measured and theoretical wave elevations at the presumed focal location, i.e., $x = 7.0$ m. In the figure, the measured carrier wave amplitude is 0.0175 m and the maximum wave crest height is 0.0501 m, leading to a ratio of 2.86, which is slightly lower than the theoretical amplification factor 3.0. Besides, the following two waves after the maximum crest are not consistent with the theoretical curve, which is also observed in figure 5 of Chabchoub et al. (2011). These differences may be due to that the approximate paddle signal did not produce the precise wave dynamics near the wave paddle or that the third-order NLS equation is not absolutely accurate to describe the wave evolution with large steepness in which effects of higher order wave components could be significant. Moreover, in contrast to the symmetric shape of theoretical wave elevation series, the measured wave elevation contains larger wave front steepness, indicating possible wave breaking.

To inspect the wave energy distributions of both the measured and theoretical wave sequences, the wave energy density spectra and the corresponding logarithmic spectra are given in Figure 5. It is observed that the energy density spectrum of measured wave is consistent with that of theoretical wave except for the insufficient amplitude. However, some differences are observed from the logarithmic spectrum. The measured wave elevation consists of low frequency, second-order harmonic and third-order harmonic components, as well as that of carrier wave frequency. It means that there also inevitably exists wave energy of higher order components in rogue waves based on the Peregrine breather model for large wave steepness.

A wavelet analysis is performed on the wave sequences to reveal the temporal-frequency variation of wave energy. A complex Morlet wavelet with central frequency $f_c = 1.0$ Hz was used as the mother wavelet function in this study and the scale factors in WT were transformed into the circular frequency values according to the following relation:

$$\omega = 2\pi \frac{f_c f_s}{a},$$

where $f_s$ is the sampling frequency and $a$ is the scale value.

Figure 3. Surface elevations at six positions along the flume. (This figure is available in colour online.)

Figure 4. Measured and theoretical time histories of wave elevations at $x = 7.0$ m. (This figure is available in colour online.)

Figure 5. Wave spectrum comparison of theoretical and measured surface elevations at 7.0 m. (a) Energy density spectrum; (b) logarithmic spectrum. (This figure is available in colour online.)
Figure 6 presents the wavelet spectra of both theoretical and measured waves. It shows that the majority of wave energy is distributed around the circular frequency of the carrier wave. In the calm region between the stable carrier waves and rogue waves, the energy is relatively low. It means that there exists a temporally low energy moment just before and after the occurrence of rogue waves. In the rogue wave region, the energy is highly centralised with the magnitude being larger and the frequency band being wider. On closer inspection, it is observed that the energy centre has shifted to the frequency level a little higher than the circular frequency of carrier wave, which is consistent with the smaller wave period at the moment the rogue wave occurs. The major difference is that at the focal moment, the measured wave elevation covers obvious higher frequency components, reaching the third-order harmonic frequency levels.

4.2. Rogue wave forces acting on the cylinder

In this study, the maximum wave height $H$ in the calibrated wave is 0.085 m and the cylinder diameter $D$ is 0.1 m. Thus, the wave force acting on the cylinder is dominant by inertia force and both the drag force and diffraction effect can be ignored considering $H/D = 0.85 < 1.0$ and $D/L = 0.1 < 0.2$.

Figure 7 shows the measured and numerical time histories of wave forces, including the horizontal wave force $F_x$ and the moment $M_y$ with regards to the point 0.25 m above still water level (SWL). Since the drag force is negligible, the numerical results were calculated based on the calibrated wave sequence and the response amplitude operator (RAO) results given by HydroSTAR. It is shown that the numerical wave force and moment agree well with the measured curves on the general trend. However, when encountering the rogue wave, the numerical results underestimate 25.0% of the wave force and 11.1% of the moment, respectively.

A fast Fourier analysis is conducted to further investigate the amplitude spectra of wave forces, see Figure 8. The amplitude spectra of numerical results have an overall agreement with that of measured results. The second-order harmonic of numerical $F_x$ (corresponding to 15.7 rad/s) is significantly smaller than that of measured value, which is not obvious for $M_y$. To make a clear analysis on the difference between numerical and measured results, the total wave forces are simply divided into four components as...
follows:

\[ F = F_1^1 + F_1^2 + F_2^2 + \text{c.c.}, \quad (11) \]

where \( F_1^1 \) represents the first-order wave force due to the first-order wave component, \( F_2^2 \) is the second-order wave force due to the second-order wave component, \( F_1^2 \) is the second-order wave force due to the first-order wave component and \( \text{c.c.} \) represents the rest higher order wave force components. It is noted that the measured incident wave sequence, which contains the second-order component as well as the first-order component (Figure 5), was adopted as an input in numerical predictions. Hence, the numerical wave forces include \( F_1^1 \) and \( F_2^2 \) components. Besides, the RAO results of wave forces were obtained without considering the second-order wave force responses which may come from the variance of the transient wet surface or the velocity quadratic term in Bernoulli equation, and thus the \( F_1^2 \) term is not included in numerical wave forces. As mentioned, in contrast to \( M_y \), the second-order \( F_x \) is more obvious and the difference between the numerical \( F_x \) and the measured \( F_x \) is relatively larger. The reason may be that the \( F_2^2 \) component comes mainly from the variance of wet surface. It is a kind of point force associated with the local region under the exact free surface, which is also indicated in Faltinsen et al. (1995). Figure 9 shows the snapshot of wave run-up at the occurrence of rogue wave. It is shown that the wave run-up is obvious and probably causes an increment of wave forces. Since the length between the reference point (0.25 m above SWL) and the free surface region is short, the second-order component of \( M_y \) is not significant in this study.

Figures 10 and 11 present the wavelet spectra of numerical and measured wave forces and moments. It is observed that the second harmonic components of numerical wave force and moment (corresponding to 15.7 rad/s) are significant only in the region of rogue waves, which are due to the second-order component of incident wave. For the measured wave force, the second harmonic components exist throughout the whole time, which are mainly the \( F_2^2 \) components. However, for the measured wave moment, the \( F_2^2 \) components are not visible due to the short length between the reference point and the free surface. Besides, in the rogue wave region, both the measured wave force and moment contain obvious high-frequency components, which may include \( F_1^2, F_2^2, F_3^2, F_4^2, \ldots \).
4.3. Coherence between rogue wave elevations and forces

To reveal the relation between the incident wave elevations and the measured wave force and moment acting on the cylinder, a coherence analysis applied to WT was performed. The wavelet coherence spectra for the wave force $F_x$ and the wave moment $M_y$ are given in Figures 12 and 13. The horizontal green dot dash lines in the figures represent the first to third harmonic frequency levels with regard to the circular frequency of carrier wave and the vertical dot dash line stay at 30 s denotes the presumed time the rogue wave occurs.

From the wavelet cross spectra, extremely large values exist around the intersections of the circular frequency and the occurrence time of rogue wave, which indicate significant contributions from both the incident wave and wave force/moment in these regions. Hence, these regions should be given great concern when investigating the wave forces/moments of rogue waves.

It is noted that the phase values in the phase angle spectra were obtained by reducing the numerical response phase operator results of HydroSTAR from the original phases of coherence. It is clear that, at the circular frequency of carrier wave, the phase angle values are nearly zero. It means that the numerical methods can well predict phase response of the first-order force/moment component. Besides, at the occurrence of rogue wave, the phase angle values are around zero within a wide frequency range. For the measured wave force, a part of the differences between the original phases of coherence and the numerical phase response values are due to the fact that among higher order force components result from lower order wave components. However, as we can see that phase differences also exit in phase angle spectrum of the measured wave moment, in which components like $F_1^2$ are ignorable. Therefore, the numerical method is not able not give accurate predictions of phase responses of rogue wave force/moment at high-frequency levels. The reason may be that the corresponding wave length of high-frequency wave components is small for the cylinder diameter and the interactions between the high-frequency wave components and the cylinder are strongly nonlinear.

Wavelet coherence spectrum is the most intuitive representation of the correlation between two signals. It can be interpreted as the local squared correlation coefficient in the time-scale plane. It is observed that for both $F_x$ and $M_y$, the coherence values in some regions are less than unity. However, at the circular frequency of carrier wave, the coherence values are all unity for the whole time and at the occurrence of rogue wave, the coherence values are all unity as well. It means that the wave force and the wave moment are well correlated with the incident wave.

5. Conclusions

To investigate the rogue waves based on the Peregrine breather model and their impacts on a vertical cylinder,
we have generated rogue waves using the Peregrine breather model in the wave flume. The wave forces acting on a stiffly mounted vertical cylinder are measured. The parameters of carrier wave were chosen to be slightly beyond the breaking threshold in order to generate a rogue wave sequence with noticeable nonlinear characteristics. The measured wave elevation have been compared with the theoretical solutions, while the corresponding wave forces (including the horizontal wave force $F_h$ and the wave moment $M_o$) have been compared with the numerical results based on the RAO results from HydroSTAR software. Both FFT and WT methods have been adopted to investigate the constitute of frequency components of the wave elevations and the wave forces. Moreover, a coherence analysis was performed to investigate the relation between the measured wave forces and the incident wave. Some conclusions are drawn as follows:

1. Being consistent with the Peregrine breather model, the maximum wave crest grows gradually as it progresses towards the focal location. The measured wave elevation contains a wider frequency band than the theoretical wave sequence;
2. The numerical wave forces are in excellent agreement with the measured wave forces except the maximum magnitudes in the region near the occurrence of rogue wave. The differences between the maximum magnitudes are due to the second harmonic wave forces induced by the first order wave elevation;
3. The linear numerical method is able to give accurate predictions of the phase response of the first harmonic force/moment, however, at high-frequency level, it is powerless;
4. The coherence analyses show that, at the occurrence of rogue wave, the wave force and moment are well correlated with the incident wave within a large frequency range.

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Disclosure statement

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